M2 JUNE 06

1) $acc = 5-2t$

$V = \int a\,dt = 5t - t^2 + C \qquad t=0, V=6 \qquad 6 = C \Rightarrow V = 5t - t^2 + 6$

at rest when $V=0 \Rightarrow 5t - t^2 + 6 = 0, \quad t^2 - 5t - 6 = 0$

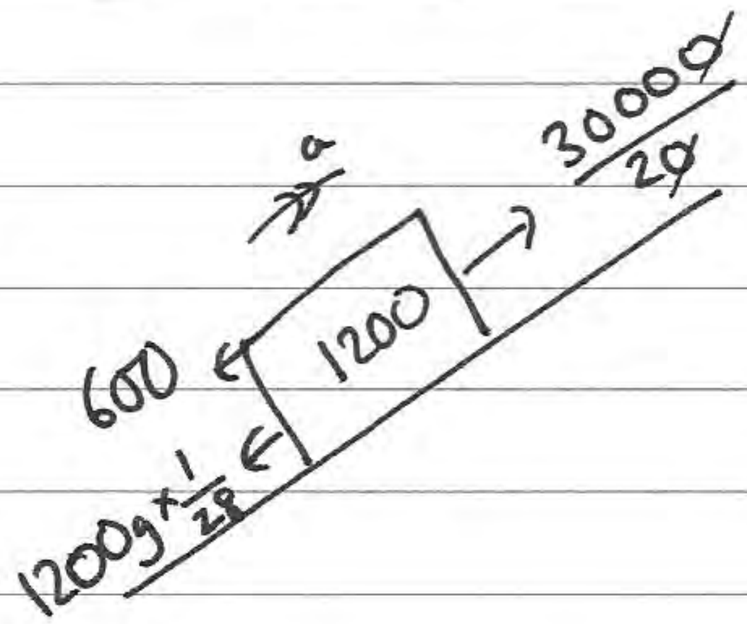
$(t-6)(t+1) = 0 \Rightarrow \underline{t = 6 \text{ sec}}$

2) $\xrightarrow{24}$

$600 \leftarrow \boxed{1200} \rightarrow \frac{P}{24} \Rightarrow \frac{P}{24} = 600 \Rightarrow P = 14400 \text{W}$

$\Rightarrow \underline{P = 14.4 \text{kW}} \quad (3 \text{ sf})$

b)



$R \uparrow = ma \Rightarrow 1500 - 600 - \frac{1200g}{28} = 1200a$

$\Rightarrow a = \frac{480}{1200} \Rightarrow \underline{a = 0.4 \text{ ms}^{-2}}$

3) Momentum before $= mu = 0.5(-30i) = -15i$

Momentum after $= mv = 0.5(16i + 20j) = 8i + 10j$

$\Rightarrow$ Impulse $= 23i + 10j$

$|\text{Impulse}| = \sqrt{23^2 + 10^2} = \underline{25.1 \text{ Ns}} \quad (3 \text{ sf})$

b) $r = 16t\,i + (20t - 5t^2)\,j$

$\frac{dr}{dt} = 16i + (20 - 10t)\,j \qquad t = 3 \Rightarrow V = 16i - 10j$

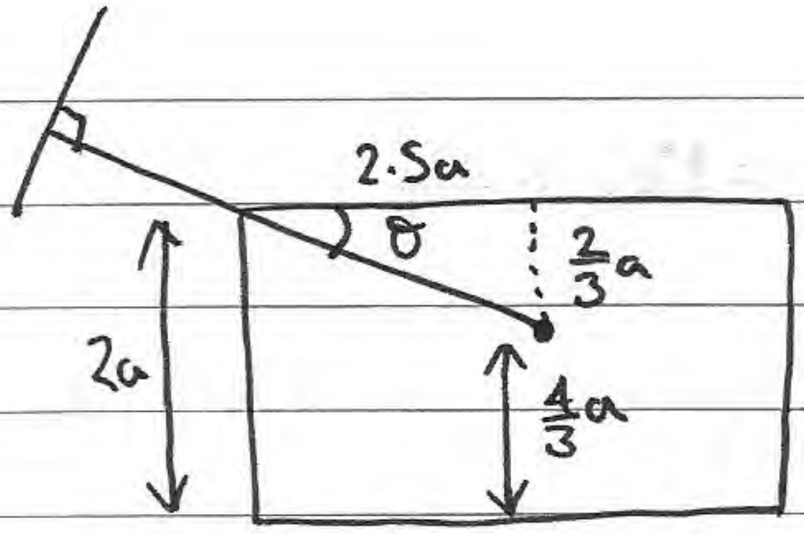
$\text{Speed} = \sqrt{16^2 + 10^2} = \underline{18.9 \text{ ms}^{-1}}$

4) $\uparrow \quad mg \times 1.5a + mg \times 1.5a + mg \times 3a + 6mg \times 3a + 2mg \times 3a = 12mg\,\bar{x}$

$30mga = 12mg\,\bar{x} \qquad \bar{x} = \frac{30}{12}a = \underline{2.5a}$

$\rightarrow \quad mg \times a + mg \times a + mg \times 2a + 6mg \times 2a = 12mg \times \bar{y} \qquad \bar{y} = \frac{16a}{12} = \underline{\frac{4}{3}a}$

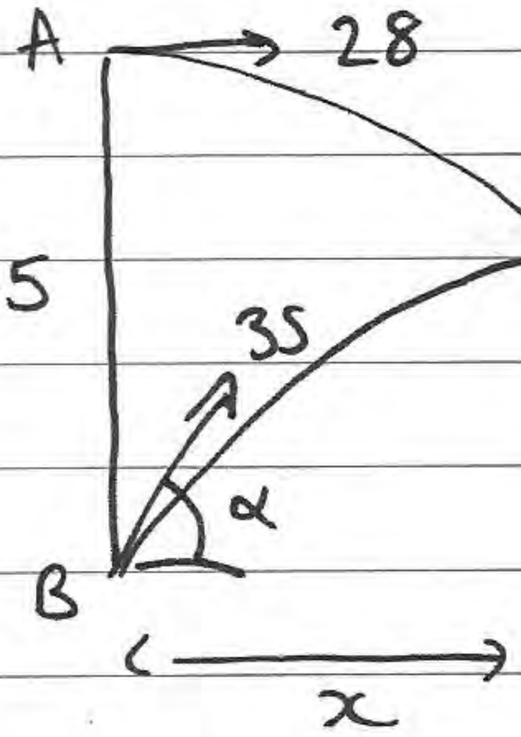
4b)



$$\theta = \tan^{-1}\left(\frac{\frac{2}{3}a}{2.5a}\right)$$

$$\theta = \tan^{-1}\left(\frac{4}{15}\right) = \underline{14.9^\circ} \quad (3sf)$$

5)



(A) $\overrightarrow{\text{Vel}} = 28 \Rightarrow x = 28t$

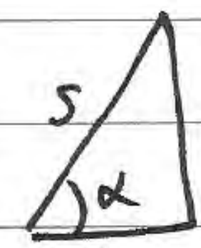
(B) $\overrightarrow{\text{Vel}} = 35\cos\alpha \Rightarrow x = 35\cos\alpha \times t$

$\Rightarrow 28t = 35\cos\alpha \times t \Rightarrow \cos\alpha = \frac{28}{35} = \frac{4}{5}$ #

(A↓) $u\downarrow = 0 \Rightarrow 73.5 - y = 4.9t^2$

$a\downarrow = 9.8$

$s\downarrow = 73.5 - y$



$\cos\alpha = \frac{4}{5}$

$\Rightarrow \sin\alpha = \frac{3}{5}$

(B↑) $u\uparrow = 35\sin\alpha = 35 \times \frac{3}{5} = 21$

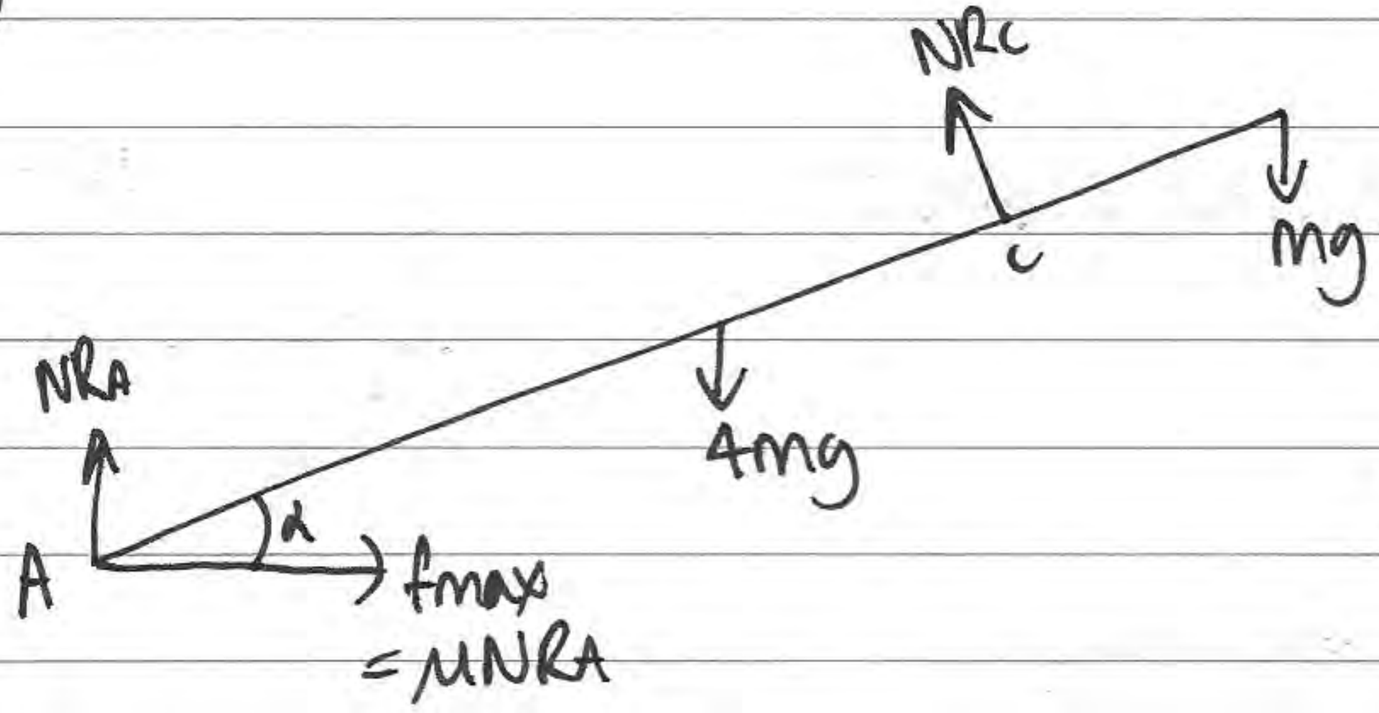
$a\uparrow = -9.8$

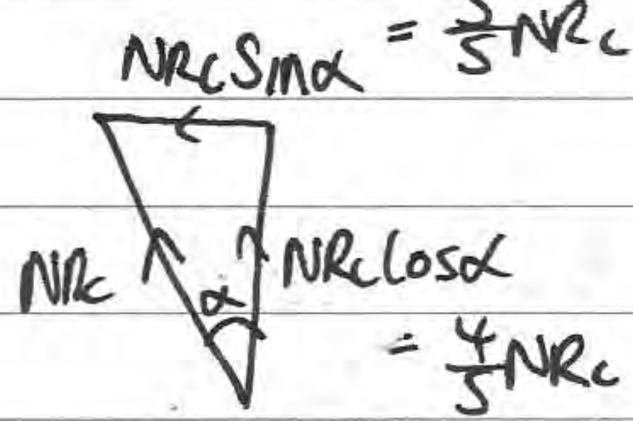
$s\uparrow = y \qquad y = 21t - 4.9t^2$ sub in (A)

$73.5 - (21t - 4.9t^2) = 4.9t^2$

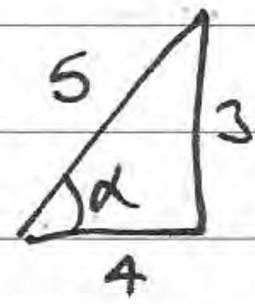
$\Rightarrow 73.5 - 21t + 4.9t^2 = 4.9t^2 \Rightarrow 21t = 73.5 \rightarrow t = \underline{3.5 \text{ sec}}$

6)





$\tan\alpha = \frac{3}{4}$



$\cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$

$\uparrow = \downarrow \Rightarrow N_{RA} + \frac{4}{5}N_{Rc} = 5mg \rightarrow N_{RA} = 5mg - \frac{4}{5}N_{Rc}$

$\rightarrow = \leftarrow \quad \frac{3}{5}N_{Rc} = f_{max} \Rightarrow \frac{3}{5}N_{Rc} = \mu N_{RA}$

$\curvearrowright \quad 4mg \times 2a\cos\alpha + mg \times 4a\cos\alpha = N_{Rc} \times 3a \Rightarrow \frac{32}{5}mg + \frac{16}{5}mg = 3N_{Rc}$

$\Rightarrow 3N_{Rc} = \frac{48}{5}mg \Rightarrow N_{Rc} = \frac{48}{15}mg \Rightarrow N_{Rc} = \frac{16}{5}mg$ #

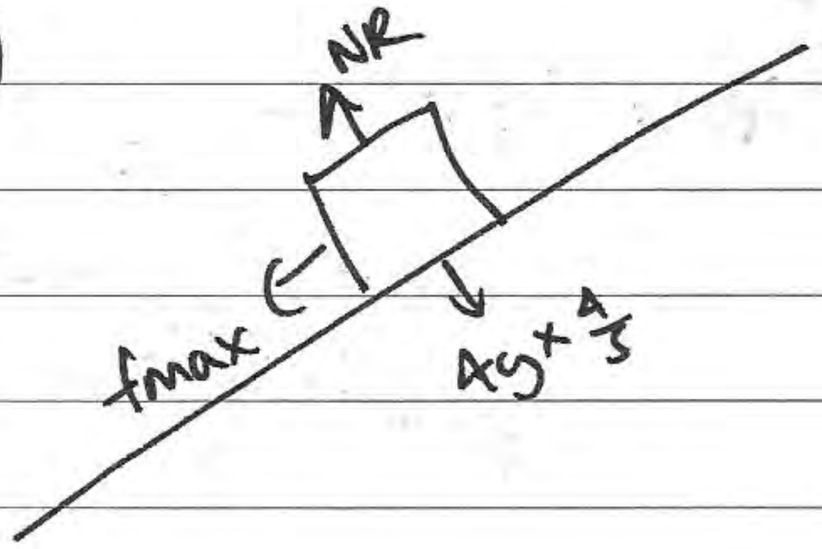
b) $N_{RA} = 5mg - \frac{4}{5}\left(\frac{16}{5}mg\right) = \frac{61}{25}mg \qquad \frac{3}{5}\left(\frac{16}{5}mg\right) = \mu\left(\frac{61}{25}mg\right)$

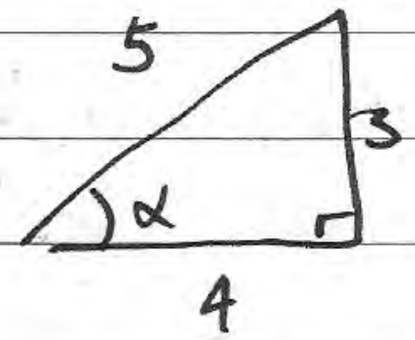
$\mu = \frac{48}{61}$ at limiting equilibrium $\Rightarrow \mu \geq \frac{48}{61}$

friction = fmax friction ≤ fmax.

c) No frictional forces at Peg.

7)





$\cos\alpha = \frac{4}{5}$

$\sin\alpha = \frac{3}{5}$

$\mu = \frac{3}{7}$

$f_{max} = \mu NR = \mu \frac{16}{5} g = \frac{32}{35} g$

Wd against friction $= f_{max} \times 2.5 = \frac{32}{35} g \times \frac{5}{2} = \frac{16}{7} g\ y$

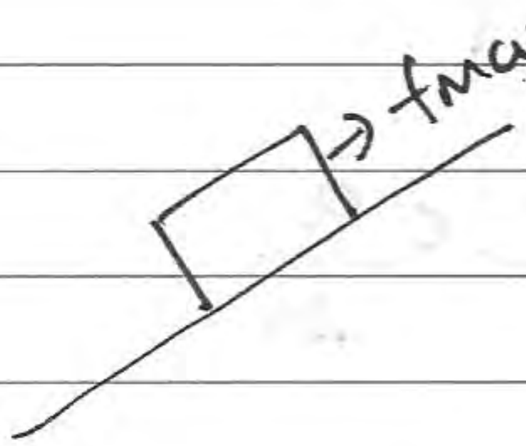
b) KE_A − Wd against friction = PE gain at B

$\frac{1}{2}(4)u^2 - \frac{16}{7}g = 4g(2.5 \times \frac{3}{5})$

$\Rightarrow 2u^2 = 6g + \frac{16}{7}g \Rightarrow 2u^2 = \frac{58}{7}g \Rightarrow u^2 = \frac{29}{7}g$

$\Rightarrow u = 6.4 ms^{-1}$ (2sf)

c)

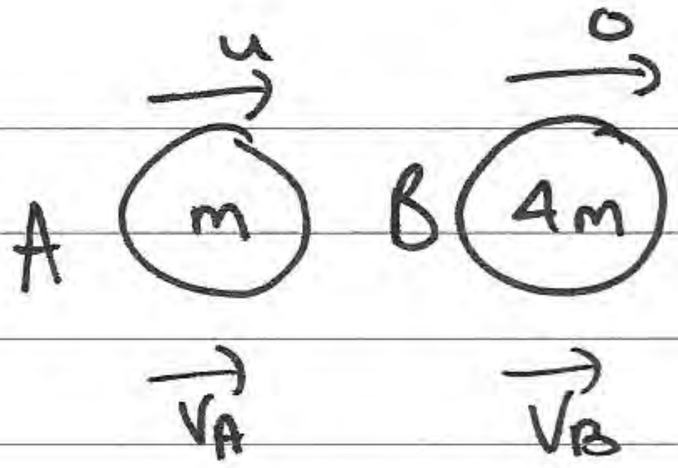


PE_B − Wd against friction = KE_A

$4g(2.5 \times \frac{3}{5}) - \frac{16}{7}g = \frac{1}{2}mv^2$

$\Rightarrow \frac{26}{7}g = 2v^2 \Rightarrow v^2 = \frac{13}{7}g \Rightarrow v = 4.3 ms^{-1}$ (2sf)

8)



$$e = \frac{V_B - V_A}{u}$$

$$eu = V_B - V_A$$

$$V_B = V_A + eu$$

CLM $\Rightarrow mu = mV_A + 4mV_B$

$\Rightarrow mu = mV_A + 4m(V_A + eu)$

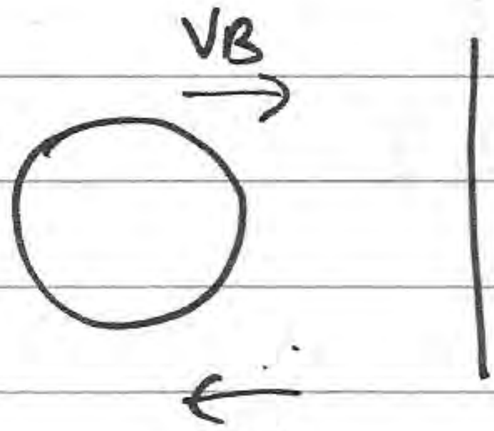
$\Rightarrow u = V_A + 4V_A + 4eu$

$\Rightarrow u - 4eu = 5V_A$ $\Rightarrow V_A = \frac{1}{5}u(1-4e)$

A is reversed

$\Rightarrow$ speed$_A = \frac{1}{5}u(4e-1)$ $\leftarrow$

$V_B = \frac{1}{5}u - \frac{4}{5}eu + eu$ $\Rightarrow V_B = \frac{1}{5}u(1+e)$



$$e = \frac{w}{V_B} = \frac{4}{5} \Rightarrow 4V_B = 5w$$

$$\Rightarrow \frac{4}{5}V_B = w$$

Since B and A collide $w > \frac{1}{5}u(4e-1)$

$\frac{4}{5}V_B > \frac{1}{5}u(4e-1)$ $\Rightarrow \frac{4}{5}\left(\frac{1}{5}u(1+e)\right) > \frac{1}{5}u(4e-1)$

$\Rightarrow \frac{4}{5} + \frac{4}{5}e > 4e - 1$

$\Rightarrow \frac{9}{5} > \frac{16}{5}e$ $\Rightarrow e < \frac{9}{16}$

$V_A = \frac{1}{5}u(1-4e)$ A is reversed $\Rightarrow V_A < 0$

Since $u > 0$ $\Rightarrow 1 - 4e < 0$ $1 < 4e \Rightarrow \frac{1}{4} < e \Rightarrow e > \frac{1}{4}$

$\therefore \frac{1}{4} < e < \frac{9}{16}$ #

c) $V_A = \frac{1}{5}u(1-4e)$ $e=\frac{1}{2} \Rightarrow V_A = -\frac{1}{5}u$

$V_B = \frac{1}{5}u(1+e)$ $e=\frac{1}{2} \Rightarrow V_B = \frac{3}{10}u$.

$KE_{Before} = \frac{1}{2}(m)u^2 = \frac{1}{2}mu^2$

$KE_{after} = \frac{1}{2}m(-\frac{1}{5}u)^2 + \frac{1}{2}(4m)(\frac{3}{10}u)^2 = \frac{1}{50}mu^2 + \frac{9}{50}mu^2$

$= \frac{1}{5}mu^2$

$KE\ lost = \frac{1}{2}mu^2 - \frac{1}{5}mu^2 = \underline{\frac{3}{10}mu^2}$